Limitations of One Dimension Modeling for the Resonance Piezoelectric Array Sensors

Dimo Rumenov Kolev and Velimira Dimitrova Todorova

Abstract – **The resonance piezoelectric tactile array sensors offer the means of measuring not only the presence of applied mechanical stress but also the local stress intensity. The problem in such kind of resonance sensors is the complex processing circuits or the real time mode of operation. It is useful to define a model for describing the sensor behavior which takes into account the array structure and uses it to diminish its disadvantages in regards to real time mode operations. For this purpose a one-dimensional mathematical model utilizing transformed form of Sauerbrey equation is under investigation for applicability for the resonance piezoelectric array sensors as well as its limitations. The aim of the current research is the substantiating the usability of this model as a base for further development of the physical model for the resonance sensor system as the quantitative solutions to be in reach.**

Keywords – **resonance, piezoelectric, array sensors, modeling**

I. Introduction

The piezoelectric resonance sensors are broadly used for detection of applied mechanical stresses because their high sensibility which is due the physical processes, which take part in the bulk of piezoelectric material, as the resonant work mode ensures low level switching threshold. This high sensibility is defined by the relation between the fundamental oscillator frequency and the piezoelectric substrate thickness or the externally brought in inertia. An applied mechanical stress will generally cause shift in the oscillator frequency. This effect is used in switch mode sensors suitable for detecting the presence or absence of applied stress, as well as in the measurement of very small mass amounts. But the nowadays tactile sensing information systems are expected to supply broader range of sensor data and thus the array type of piezoelectric resonance sensors are introduced.

A. Methods for forming the array structures

The discrete piezoelectric resonance sensors are used for detecting the presence of the external mechanical stress and even can be utilized for obtaining the average volume of the applied force. The high sensibility is due to the strong relationship between the fundamental frequency and the other parameters of the resonance system. If there is a slight change in one of resonance parameters, there will be an equivalent change in the fundamental frequency [6]. Therefore, the fundamental frequency shift can be applied for determining the average applied force [1] through the basic equation for the piezoelectric medium:

$$f_0 = \frac{1}{2d}\sqrt{\frac{Y}{\rho}} \tag{1}$$

where d is the thickness of piezoelectric substrate, f_0 – frequency of the fundamental, Y – Young's modulus and ρ – material density.

But this information is not enough to represent the actual shape of the object surface (fig. 1). In this case the local distribution of the applied stress is required to be obtained and thus the need of utilizing a structure of array type.

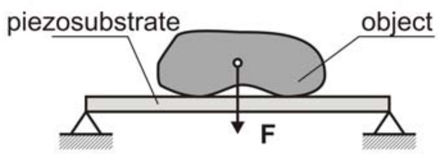


Fig. 1. Loaded piezoelectric substrate

When the resonator is loaded (i.e. some external mechanical stress is applied), there is a change in the substrate thickness, which causes a proportional shift in the resonance frequency too (fig. 2). If the condition $\Delta d << d$ is fulfilled (which is usually true in piezoelectric sensor applications) then it can be deduced that [6]:

$$\frac{\Delta f}{f_0} = -\frac{\Delta d}{d}, \tag{2}$$

where Δd is thickness change.

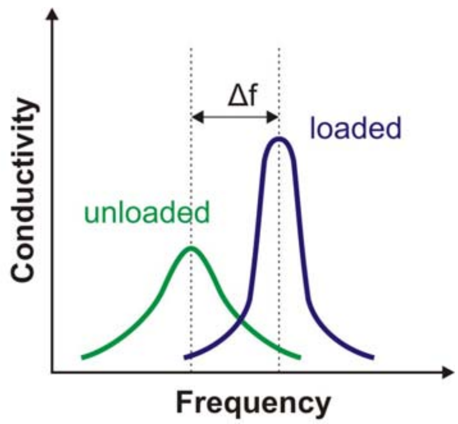


Fig. 2. Dependency of the fundamental frequency from the applied mechanical stress

The shift in the fundamental frequency of quasi-discrete resonators can give the distribution of applied mechanical stress and the contours of the object causing this stress can be determined [1].

D. Kolev is with the Department of Electronics, Faculty of Electrical Engineering and Electronics, Technical University – Gabrovo, 4 Hadji Dimitar Str., 5300 Gabrovo, Bulgaria, e-mail: uplift7@mail.bg

V. Todorova is with the Department of Electronics, Faculty of Electrical Engineering and Electronics, Technical University – Gabrovo, 4 Hadji Dimitar Str., 5300 Gabrovo, Bulgaria, e-mail: vili@tugab.bg

Piezoelectric resonance array structure can be achieved with hybrid construction of membrane and multiple miniature resonators with same properties laid on it. But the resonators will have their tolerances and the membrane will cause an influence over the operational mode of the resonators with its elastic properties.

Better solution is to form miniature resonators on the one common piezoelectric substrate. If resonators are formed as pair of electrodes with same shape (fig. 3), it will insure the exact same properties for each of the resonators from the array field.

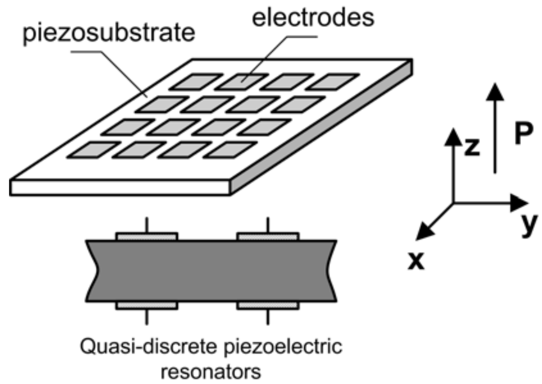


Fig. 3. Piezoelectric resonance array sensor
P is the vector of piezoelectric substrate polarization

The separation between the resonators (or the array nodes) is achieved with the utilization of the piezoelectric medium anisotropic properties and the polarization along Z axis of the piezoelectric substrate. The resonators are excited with the fundamental frequency of the thickness wave resonance (along Z axis) which will guarantee the high fading coefficient in the other directions.

Such kind of array structure has some drawbacks as with the increase of its dimensions (i.e. increase in the number of resonator nodes which the array is composed from) the wiring system becomes more complex and the parasitic capacitance increases. Furthermore the data processing circuits become increasingly complicated which leads to applying the new designs [1] of the array field (fig. 4).

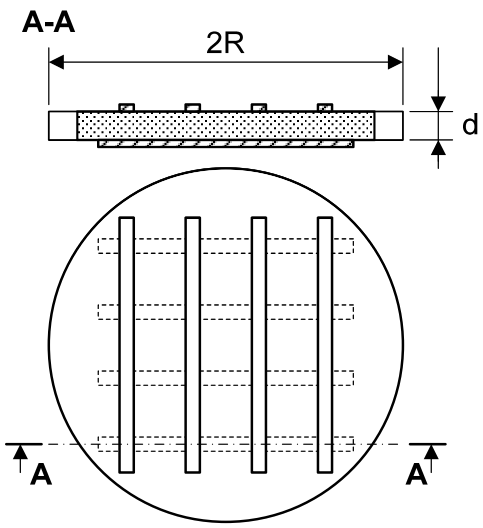


Fig. 4. Alternative array design

The alternative array configuration is obtained through the electrode design [1] in which the array nodes are created at the crossing of the electrode systems from the two planes of piezoelectric substrate (fig. 4). In this case the oscillator separation is achieved via the usage of the thickness acoustic waves and polarization of the substrate along the Z axis as the longitudinal share of distortion is ignored. The structure can be considered quasi-array like because of its operational principle as the array is scanned consecutively as the quasi-discrete resonators (considered as separate) are excited one after another. Thus in the given moment of time there is only one resonator that is operational.

The classical scanning method is to consecutively excite every single array node and thus, judging on the shift in the fundamental frequency the local distribution of applied mechanical stress is obtained. But this is true only when the stress time duration exceeds the time required for array scanning and data processing. When this condition is not met only partially true tactile information is obtained. The sensibility and spatial resolution of the sensor array is defined by the number of array nodes and their distribution across the substrate.

Naturally the sensibility and the resolution will increase with the increase in the number of nodes, but it is posing a problem when the array field is consecutively scanned as the time for the processing data is accordingly increased. The problem can be partially solved by utilizing parallel scan of several array nodes (fig. 5) with the assumption that in the piezoelectric medium are excited only thickness waves or the longitudinal and radial waves have relatively high fading coefficient.

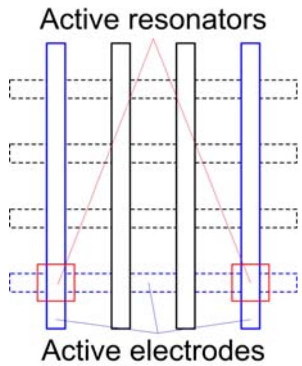


Fig. 5. Parallel scanning of an array field

In reality when exciting thickness waves in piezoelectric medium there are also longitudinal waves, i.e. the separate oscillators exercise some degree of influence on each other and therefore when parallel scanning is used they can not be considered to be separated entities on the piezoelectric substrate. They should be rather presented as sophisticated system of oscillators with mechanical connection fulfilled by the piezoelectric substrate between them.

B. One Dimensional Model Basics

Parallel scanning of the array nodes can be used to partially solve the problem with real time mode operation of the sensor. If all of the column electrodes (fig. 4) are excited and only one from the row electrodes is excited in the given moment of time, then a parallel scan of the array can be utilized as the array field is scanned row by row. The resonator nodes along the row can be combined in string structure on which one dimensional model can be based.

When mathematical model of resonance array sensor is being created it is worth considering the use of electromechanical analogy [3] as base for the model. The direct and converse piezoelectric effects are relatively thoroughly described for the cases of simple piezoelectric sensors (switch type sensors), as the equivalent circuit, is used [3]. The frequency change in this case is as follows:

$$\frac{\Delta f}{f_0} = \frac{i}{\pi Z_q} Z_L , \quad (3)$$

where f_0 is the frequency of the fundamental, Z_q – acoustic impedance of the material, Z_L – load impedance.

Butterworth-van-Dyke equivalent circuit [7] is applicable when numerically determining the parameters of piezoelectric process using small load approximation. The equivalent circuit also considers the influence exercised by the electrode configuration as well as the mounting method and the form used for piezoelectric substrate in the sensor system. But when dialing with the multitude array nodes the equivalent circuit should be split on multiplicity of oscillators. For the different combination of active electrodes the corresponding equivalent circuit consisting of oscillators, connected in series or in parallel to each other should be created. This path of modeling is not convenient as each combination of active electrodes will be described with different equivalent circuit.

The more convenient way is to treat the array nodes as quasi-discrete oscillators bound mechanically by piezoelectric substrate. This assumption is accepted as there are localizations in the energy distribution of the oscillator system [2]. In this case it is suitable to use the Sauerbrey equation [4, 6] to solve the problem with the each of the quasi-discrete oscillators.

The quasi-discrete oscillators can be presented via Sauerbrey equation, which disregard the influence of electrode configuration. The mechanical stress is determined via inertia ($\sigma = -\omega^2 u_0 m$, where u_0 is the amplitude of oscillation and m is the (average) mass per unit area). Considering the result, then:

$$\frac{\Delta f}{f_0} \approx \frac{i}{\pi Z_q} \frac{-\omega^2 u_0 m}{i\omega u_0} = -\frac{2f}{Z_q} m , \quad (4)$$

This approach defines the mechanical stress as the inertial function, as it is assumed that the load has the same acoustic properties as the substrate and is homogenously distributed over the substrate surface [5], in which way the load is treated as substrate extension.

The fundamental frequency shift of every resonator can be presented via Sauerbrey equation with the assumption that each of the resonators has constant mass m (they are constructed on the same substrate), to which the external mechanical stress is adding additional inertia. Therefore the constant for the mass m in Eq. 4 should be replaced with the variable average mass $m_S = m + m_{EXT}$, where m_{EXT} is the mass, representing actually the inertia brought in by the external force F and accordingly (4) will transform to:

$$\frac{\Delta f}{f_0} = -\frac{2f}{Z_q} m_S , \quad (5)$$

Then the behavior of resonators formed along the lower electrode can be described with one dimensional model. Every resonator can be described with variable mass m_S and connected with his neighbors via massless springs with length r (distance between the parallel electrodes) and stiffness k – fig. 6. Actually the stiffness k is replaced with the piezoelectric constant k_{31} which is electromechanical constant for the longitudinal acoustic waves in the polarized along Z axis piezoelectric substrate (which is used to form the sensor array).

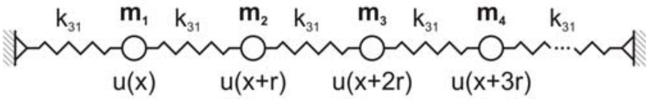


Fig. 6. One dimensional model for sensor row

In this case it is obvious that in the specific moment of time when there is no external stress bringing in additional inertia over the sensor, the respective masses of the resonators are equal to each other – m (same piezoelectric substrate and same resonance frequency). When the external stress is applied then every resonator mass will became an indication for the externally applied force. Then the adapted form of wave equation with conjunction with Hooke's law can be used to determine the interaction between the resonators – for instance when determining the mass m_{S2} of the second resonator the following expression is deduced:

$$\frac{\partial^2 u(x+r,t)}{\partial t^2} = \frac{k_{31}[u(x+2r,t) - 2u(x+r,t) + u(x,t)]}{m_{S2}} , \quad (6)$$

where $u(x)$ measures the distance from the equilibrium of the mass situated at x point, t – time.

Therefore the mass can be determined as:

$$m_{S2} = \frac{k_{31}[u(x+2r,t) - 2u(x+r,t) + u(x,t)]}{\frac{\partial^2 u(x+r,t)}{\partial t^2}} . \quad (7)$$

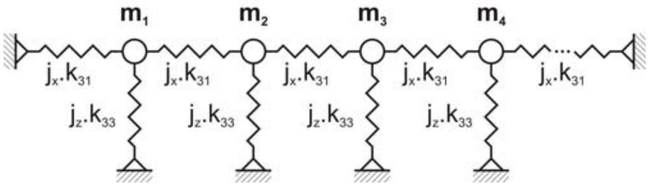
C. Limitations of One Dimensional Model

The assumptions for which the proposed model is accurate are as follows: conditions for small-load approximation hold; there are neither compression waves nor flexural contributions to the displacement pattern, there are no nodal lines in the plane of the resonator; all stresses are proportional to strain, the lows of linear acoustic hold; piezoelectric stiffening may be ignored.

These conditions not always can be kept, for instance if the volume of the applied external force surpasses the value needed only for the compression of the substrate thickness, then substrate itself will bend which will make the one dimensional model obsolete.

Also the model does not reckon with the spatial distribution and sizes of the electrodes, which also should be taken into account. The zero-displacement boundary condition on the outer edges of the model (defined by the sensor mounting) will help to solve numerically (7), applying the finite element method. But this boundary condition is only true for this particular sensor mounting.

For these reasons it is more appropriate to adopt form of quasi 2D (two dimensional) model (fig. 6).



k_{33} – electromechanical constant for thickness acoustic waves
Fig. 6. Quasi 2D model

In this form of the model two additional functions are subjoined. The function j_x takes heed of the electrode sizes and spatial distance between them and the function j_z, which deals with the condition of small-load approximation. The values of the function j_z are zero when the conditions of small-load approximation hold and have value different than zero when these conditions are broken. In this way when the conditions of the approximation are unbroken the model is reverted to the one dimensional one.

Although the model can help to speed up the scanning process it is not paying attention to interaction between the different rows of the array, which require additional expansion of the model.

II. Conclusion

The sensor system is aimed at not only the detection of the mechanical stresses but also for the determination of the stress vector and the stress force. This can be achieved by using array nodes, formed on common piezoelectric substrate. The determination of mechanical stress is done by measuring the resonance frequency change of the separate nodes. The frequency change is in direct dependency on the local change of substrate thickness and Sauerbrey equation can be locally used after required transformation. Because of the fact that the separate oscillators are bounded together via the substrate, the influence between oscillators has to be taken into account.

The proposed one dimensional model for solving in quantitative way the mathematical model for resonance sensor array needs deeper development as the expressions in Eq. 5 and Eq. 7 need further refinement to allow the construction of quasi-2D model. Additional simulations and experimental researches are needed to confirm the plausibility and the reliability of the proposed model.

The merge of the methods needs the clarifying the assumptions, admissible for the researched system as well as additional experimental researches that can confirm the applicability of the developed hybrid modeling system.

References

[1] G. Murali Krishna, K. Rajanna (2004). *“Tactile Sensor Based on Piezoelectric Resonance”*. IEEE Sensors Journal, vol. 4, №. 5, 2004, p. 691-697.

[2] В. В. Малов. *Пьезо-резонансные датчики*. Изд. “Энергия”, 1978, 248 p.

[3] R. N. Thurston (1984). "36". in C. Truesdell. *“Mechanics of Solids”*. 4. Heidelberg: Springer. p. 257.

[4] V. V. Borovikov, R. A. Dialnyan and I. M. Shmyt’ko (1987). Sov. Phys. Tech. Phys. 32: 325.

[5] E. Benes, M. Groschl, W. Burger, and M. Schmid. *“Sensors based on piezoelectric resonators”*, Sens. Actuators A, vol. 48, pp. 1–21, 1995.

[6] T. Nakamoto and T. Moriizumi (1990). *“A Theory of a Quartz Crystal Microbalance Based upon a Mason Equivalent Circuit”*. Jpn. J. Appl. Phys. 29: 963.

[7] W. P. Mason. *“Electromechanical transducers and wave filters”*. Prinston, New Jersey, 1948. 419 p.